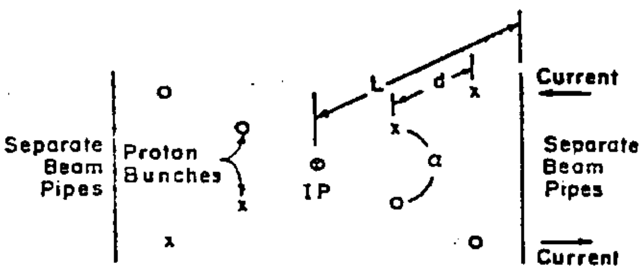
LONG RANGE BEAM-BEAM EFFECTS

FERMILAB-Conf-84/15-THY

P.L. Morton* and J.F. Schonfeld**

We summarize and discuss here the results of ...veral calculations concerning the incoherent ...eraction between SSC beams within an interaction region (IR), but away from the crossing point (IP). We shall assume that both beams consist of protons. Thus, the beams cross at a small but nonzero angle (α) and have no shielding between them only for a limited distance (L) on either side of the crossing point (see figure).



One IR, looking down

The following general definitions and observations will be used liberally:

The distance between bunches in a beam is d. Thus, a bunch travels a distance d/2 between points at ...ich it is at the same azimuth as a bunch from the opposing beam. Assuming no deviation from the straight-line paths shown in the figure, i.e., on the closed orbit in the limit of zero beam-beam ...teraction, the distance between such passing bunches ...s $-\alpha|s|$, where s is the (algebraic) distance to the IP.

The beta functions in a field-free IR depend on s according to $\beta_i = \beta_i^*[1+(s/\beta_i^*)^2]$, where i is x or y. Since typically $L \gg \beta_i^*$, β_i is thus close, over most of the IR, to s^2/β_i^*. The size of a bunch along the i-axis depends on azimuth according to $\sigma_i(s) = \sigma_i^*[\beta_i(s)/\beta_i^*]^{1/2} = s[\sigma_i^*/\beta_i^*]$.

Closed-Orbit Distortion

The horizontal closed orbit distortion in either beam at azimuth s due to long range beam-beam interactions in an IR centered at s=0 is given by (cf. Ref. 1, eq.(2.94))

$$x_c = -\frac{\beta_x^{1/2}(s)}{2\sin\pi\nu_x}\int_{-L}^{+L} d\bar{s}\ \beta_x^{1/2}(\bar{s})\left(\frac{4e^2N}{E\alpha|\bar{s}|d}\right)\cos\left[\int_0^s \frac{ds'}{\beta_x(s')} - \int_0^{\bar{s}} \frac{ds'}{\beta_x(s')} - \pi\nu\right] \quad . \quad (1)$$

In formulating this expression, we have assumed that the distance between opposing bunches that pass at azimuth $\bar{s}$ is the noninteracting value $\alpha|\bar{s}|$. We have also taken the liberty of smearing the charge that one beam sees in the opposing beam into a smooth unbunched distribution. We have also assumed that the reference point s is not contained in the IR in question, and (see below) that there are no exceptional gaps in the beam.

When $\bar{s} \gtrsim \beta_x^*$, which is the case for most of the $\bar{s}$-integration in (1), the second integral in square brackets is approximately $(\pi/2)$ sgn s. Therefore, it follows from (1) that

$$x_c/\sigma_x(s) = \frac{-\cos\left[\int_0^s \frac{ds'}{\beta_x(s')} - \pi(\nu+\frac{1}{2})\right]}{\sin\pi\nu_x}\left\{\left(\frac{e^2N}{\alpha\sigma_x^*E}\right)\left(\frac{4L}{d}\right)\right\} \quad . \quad (2)$$

For a representative estimate of the quantity in curved brackets, take $N=10^{10}$, $\alpha=50\mu$rad, L=80m, d=6m, $\sigma_x^*=7\mu$, E=20 TeV. Then

$$\left\{\left(\frac{e^2N}{\alpha\sigma_x^*E}\right)\left(\frac{4L}{d}\right)\right\} = 10\% \quad . \quad (3)$$

Thus long range effects in several IR can cause the beams at the IP in a distinct IR to miss, or at least to hit very much off center, if no corrective measures are taken.

Equation (1) is to be modified in a straightforward way when s is contained in the IR whose long range effects are being evaluated. When this modification is taken into account, one finds that an IR contributes zero to the closed-orbit distortion (although not to its derivative) at its own IP (although not at the IP's in other IR's).

Nonlinearities

Let I_y and θ_y represent the vertical canonical action and angle for the linear collision-free part of the equations of motion of a proton in one of the two storage rings. Then the long range beam-beam interactions in a single IR make the following contribution (up to an overall phase; for small α; and to leading order in powers of I_y) to the Fourier coefficient of $\exp i(N_y\theta_y + 2\pi ns/C_y)$ in the proton's complete Hamiltonian,

$$C^{-1}\int_{-L}^{+L} d\bar{s}\left(\frac{e^2}{E}\right)\left(\frac{2N}{d}\right)(2s\alpha)^{-|n_y|}(2I_y\beta_y)^{|n_y|/2}$$

$$= C^{-1}\left(\frac{2e^2N}{E|n_y|}\right)\left(\frac{4L}{d}\right)\left[\frac{1}{\sqrt{2}}\left(\frac{\sigma_y^*}{\alpha\beta_y^*}\right)\right]^{|n_y|}\left[\frac{I_y}{\langle I_y\rangle}\right]^{|n_y|/2} \quad \text{for } |n_y| \text{ even,}$$

$$0 \quad \text{for } |n_y| \text{ odd} \quad . \quad (4)$$

where C is the storage ring circumference, and $\langle I_y\rangle$ is equivalent to $\sigma_y^2/\beta_y - (\sigma_y^*)^2/\beta_y^*$. Expression (4) is a nonlinear generalization of the formulae derived in Ref. 2. This expression is valid as long as the beam separation is much larger than the beam width, i.e., $\sigma_x \ll |s|\alpha$, i.e., $\sigma_x^* \ll \alpha\beta_x^*$; and as long as $[\langle I_y/\langle I_y\rangle]^{1/2} \ll \sqrt{2}[\alpha\beta_y^*/\sigma_y^*]$. As in the preceding section, we have implicitly debunched the charge distribution in the opposing beam. The same expression, with y replaced by x, describes the coefficient of $\exp i(n_x\theta_x + 2\pi ns/C)$.

Expression (4) is to be compared with the magnitude of the contribution due to the primary beam-beam collision at the center of the IR. Once again ignoring the bunching of the opposing beam, we obtain this magnitude by forming the combination

$$C^{-1} \frac{(\Delta\nu)\langle I_y \rangle}{|n_y|} \frac{R_{|n_y|}(r)}{r\, D'(r)} , \qquad (5)$$

where the functions R and D' have been tabulated by Month (Ref. 3, Figures 1 and 2b) for $|n_y| = 6, 8, 10, 12,$ and 14. The variable r is equal to $[I_y/\langle I_y \rangle]^{1/2}$. $\Delta\nu$ is the primary beam-beam tuneshift.

In the accompanying table, we give rough values for the ratio of expression (4) to expression (5), for different values of r, and different values of $|n_y|$, for the parameter values given in the preceding section, supplemented by $\Delta\nu = .002$. The blanks correspond to values of $R_{|n_y|}$ that are too small to be read from the associated graph in Ref. 2. It seems clear that in this case the short-range nonlinearites dominate, even though the long-range interactions can produce a linear tuneshift

$$\delta\nu = \frac{1}{2\pi} \int_{-L}^{+L} d\bar{s} \left(\frac{e^2}{2\pi E}\right)\left(\frac{2N}{d}\right) \frac{1}{\alpha^2 \beta_y^*} = \left(\frac{e^2 N}{2\pi E}\right)\left(\frac{4L}{d}\right) \frac{1}{\alpha^2 \beta_y^*} , \qquad (6)$$

per IR, that can be comparable to $\Delta\nu$. For the parameters used in the table, $\delta\nu$ is $\approx .001 \approx \Delta\nu/2$. (In experiments at SPEAR,[4] with one bunch per beam and no short-range crossings, long-range nonlinearities were found to be negligible as long as the long-range distance of closest approach was greater than about three-halves times the largest transverse beam dimension.)

Table: Ratio of (4) to (5)

r	$\|n_y\|=6$	$\|n_y\|=8$	$\|n_y\|=10$	$\|n_y\|=12$
1	.002			
2	.01	.0003		
3	.04	.004	.0004	
4	.2	.02	.002	.0002

"Pac-Man" Effect

A substantial long range tuneshift and/or long range closed orbit distortion becomes problematical when the SSC beams contain long (>>d) gaps. Such gaps were considered at this Workshop as a means of keeping high-intensity beam away from an abort system between the time the system turns on, and the time at which it is fully powered.

Imagine now a bunch that enters an IR when part of a gap in the opposing beam is also passing through the same IR. This bunch passes fewer than the full 4L/d opposing bunches that it would pass if the gap were not present. Thus this IR contributes less to the long-range tuneshift of this bunch, and to its closed orbit distortion, in other IR's, than what it would contribute if the gap were not present. This IR also now makes a nonzero contribution at its own IP to the closed-orbit distortion of this bunch.

One can imagine that if these changes, which distinguish such a bunch from the typical bunch, were not specially corrected, then such a gap-encountering bunch might thereby eventually become significantly degraded, either because of repeated unstable off-center close beam-beam collisions, or because its tune is shifted from the narrow range within whic head-on high-current beam-beam collisions are stable. In this way, a gap in one beam might create or widen a gap in the opposing beam, which could in tur eliminate or degrade more bunches in the first beam and so on. One is reminded of the dot-eating protagonist in the electronic arcade game "Pac-Man."

Several possible cures for this problem were mentioned at this Workshop: (1) Separate tune and orbit control for separate bunches. (2) Use of non-superconducting and/or wide-aperture magnets in the vicinity of the abort system. In this way, one might be able to forego a gap in the beam, because then beam loss during the onset of abort might not be so damaging. (3) Development of novel fast abort methods, perhaps using RF Kickers.

Footnotes and References

* Stanford Linear Accelerator Center.

** Fermi National Accelerator Laboratory.

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